**Heisenberg’s uncertainty principle and the diffraction of light**

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**Abstract**

To test the hypothesis that the diffraction of light is a macroscopic manifestation of photons obeying Heisenberg’s uncertainty principle, I designed an experiment to investigate whether the inverse correlation between the uncertainties of position and momentum of elementary particles asserted by Heisenberg’s uncertainty principle can be derived from the relationship between the dispersion of photons diffracting through a single narrow slit and the slit’s width. After analyzing the results of my experiment with respect to Heisenberg’s uncertainty principle, I was able to establish the value of Planck’s constant as ℎ ≈ (7 ± 3)×10 −34 J s, which only deviates from its original value of ℎ ≈ 6.63×10 −34 J s by 4.2%. This confirmed my hypothesis within the acceptable margin of error of my experiment, and thus proved Heisenberg’s uncertainty principle to be a more complete explanation for why the diffraction of light occurs than Huygen’s principle as used by most physics textbooks. This research paper achieved grade A for being awarded 31/34 marks by the International Baccalaureate Organization.

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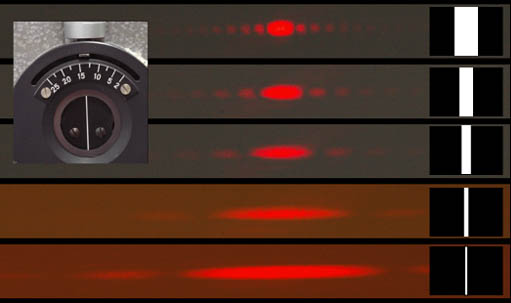
**1. Introduction**

*1.1 Background*

Most textbooks explain why diffraction occurs when waves encounter an obstacle with Huygen’s principle, which states that “each point on a wavefront acts as a point source that emits spherical wavelets, and at any later time, the total wavefront is the envelope that encloses all of these wavelets” *(University of Colorado Boulder)*. However, with reference to *Figure-1* and *Figure-2 (Georgia State University, Single Slit Diffraction)*, Huygen’s principle of wavefront does not explain why the diffraction pattern of a coherent monochromatic light source passing through a single slit becomes more dispersed as the slit narrows.

*Figure-2*: Diffraction observed in real life, diffraction pattern becomes less dispersed when the slit is wider. *(Georgia State University, Single Slit Diffraction)*

*Figure-1*: Diffraction according to Huygen’s principle of wavefronts, diffraction patterns should become more dispersed when the slit is wider.

After reading about Heisenberg’s uncertainty principle in *Topic-12* of the HL physics syllabus, I realized that it potentially offers an exquisite explanation towards why the diffraction of light occurs that accounts for aforementioned phenomenon. This lead to the research question: “***Does the relationship between the width of a single slit and the dispersion of the diffraction pattern formed when monochromatic coherent light passes through it demonstrate Heisenberg’s uncertainty principle?***”

*1.2 Theory*

The uncertainty principle asserts that “there is a fundamental limit to the precision with which certain pairs of physical properties of a particle, such as position and momentum, can be known simultaneously, which the more precisely the position of some particle is determined, the less precisely its momentum can be known and vice versa” *(University of Sydney)*. Heisenberg’s uncertainty principle for position and momentum is mathematically expressed as:

, where *h* is Planck’s constant and the variables and stands for the uncertainties of a simultaneous measurement of position *x* and momentum *p* of a quantum object.

Heisenberg’s uncertainty principle becomes relevant to the diffraction of light when we take the particle-wave duality of light into account, which allows light as particles to theoretically possess positions and momenta. The simultaneous uncertainty of position and momentum for photons passing through a narrow slit can be derived by considering *Figure-3*, where the *y*-axis measures the linear position of a photon perpendicular to the slit, and *py* is the photon’s component momentum along the *y*-axis. The range of possible *y* positions of photons passing through the slit is restricted within the width of the slit *d*, hence a narrower slit would mean that we are less uncertain about the linear position *y* of any individual photon passing through the slit. Simultaneously, the same photons will exit the slit at different angles of *θ* due to the range of different component momenta *py* they possess. Given that the relationship between *θ* and *py* is positive within the domain of , a diffraction pattern that is more dispersed along the *y*-axis would mean that we are more uncertain about the component momentum *py* of each individual diffracting photon.

*Figure-3*: Geometry of single slit diffraction in relation to a photon’s momentum.

**2. Hypothesis**

*2.1 Hypothesis based on theory*

My hypothesis is that the phenomena of diffraction patterns becoming more dispersed as the slit narrows is a direct result of photons obeying Heisenberg’s uncertainty principle. With respect to the stated theory, the following line of reasoning shows how Heisenberg’s uncertainty principle can cause light to diffract after entering a narrow slit and demonstrate this phenomena:

When a photon enters a slit that is narrow enough to reduce to the point that the product of and drops to fundamental limit asserted by Heisenberg’s uncertainty principle, of the photon will be forced to increase to accommodate Heisenberg’s inequality [1]**[[1]](#footnote-2)**. Given that light will always travel at the same speed in the same medium, this increase in can only result the photon having a higher probability of directionally deviating from its previous trajectory, causing it to have a higher probability of exiting the slit at a different angle from the angle that it entered. An observable stream of light would contain a great number of photons, hence every photon’s increased tendency to deviate from the angle that it enters the slit from will statistically manifest as the stream of light dispersing after exiting the slit, which causes the phenomenon that we call diffraction. Hence, passing light through narrower slits will cause a greater increase of , which in turn causes a greater probability for individual photons to deviate from its previous trajectory at a greater angle and resulting in a more dispersed diffraction pattern to be observed at a constant distance.

*2.2 Mathematical interpretation of hypothesis*

First, it should be noted that the original inequality [1] describes the behavior of the uncertainties of position and momentum in three-dimensional space, but the variables and defined in the introduction are one-dimensional, as they are only measured along the *y*-axis on *Figure-3*. Hence the fundamental limit of the product of and should be some unknown constant other than stated in [1].

Furthermore, it should be realized that the tradeoff between the uncertainties of momentum and position due to Heisenberg’s uncertainty principle will only occur when the product of the pair of uncertainties exceed their fundamental lower-bounds asserted by the inequality. Hence the product of and has to be exactly equal to the fundamental limit for diffraction to happen as hypothesized.

Based on the two aforementioned points, the anticipated relationship between and for diffracting photons is:

, where *k* is the unknown constant representing the fundamental limit of the product of the one-dimensional uncertainty of the diffracting photons’ position and momentum.

Subsequently, the hypothesis can be verified by investigating whether holds true for diffracting photons through experiment.

*2.3 Literature*

Werner Heisenberg himself proposed in his 1930 publication ‘*Principles of Quantum Theory’* that for electrons passing through a narrow slit, the product of one-dimensional uncertainties of position *x* and momentum *px* satisfies the inequality:

*(Heisenberg et al. 14)*

Richard Feynman later explained in one of his lectures that this inequality is obtained by assuming that most diffracting electrons will fall within the central maxima of the diffraction pattern formed *(Belle)*, and Louis de Broglie’s hypothesis that

*(Belle)*

holds true for electrons, where *p* is the momentum of the electron, is the wavelength of the electron, and *h* is Planck’s constant.

Although photons and electrons are different particles, they both possess particle-wave duality and therefore will diffract following the same principles when passed through a narrow slit, which means that they should share the same uncertainty relation during diffraction. Hence if we can prove for the anticipated relation that:

through experiment, it would further evidence that the diffraction of light does exhibit Heisenberg’s uncertainty principle.

**3. Experiment Design**

*3.1 Variables*

An experiment is designed to study how the dispersion of diffraction patterns from a monochromatic coherent light source vary when the slit width *d* changes. The width of the central maximum *w* and the maximum intensity *I0* of each distinct diffraction pattern is measured to evaluate its magnitude of dispersion.

The ultimate objective of this experiment is to determine whether the relationship between and of diffracting photons conforms with the hypothesis. of the diffracting photons is derived from the proximate independent variable *d* changed in this experiment, while of the diffracting photons is derived by studying the dispersion of diffraction patterns formed at different slit widths through the proximate dependent variables *w* and *I0* measured in this experiment.

The controlled variables for this experiment are tabulated on *Table-1*.

|  |  |  |  |
| --- | --- | --- | --- |
| *Table-1*: Variables controlled during experimental measurements of *I0* and *w* | | | |
| Symbol | Definition | Control value | Reason for control |
| *D* | **Perpendicular Distance** between the slit and the wall where the diffraction pattern is projected. | 1±0.005 m | The diffraction pattern changes independently of *d* when *D* changes. (see [8]**[[2]](#footnote-3)**) |
| *λ* | **Wavelength** of the light source used. | 632.8 nm  (Helium-neon laser) | The diffraction pattern changes independently of *d* when *λ* changes. (see [8]) |
| *θi* | **Incident angle** of light source on the slit. | 90°±0.5° | The diffraction pattern would be distorted if *θi* ≠90°. |
| *P* | **Output power** of the laser source. | 1 mW | *I0* is dependent on *P*. |
| *La* | **Ambient** luminosity. | ≤2±1 Lux | Ambient light interferes with *I0* measurement. |

*3.2 Experiment setup*

*3.2.1 Description of setup*

*Figure-4*: Simplified diagram of the experiment setup.

**1.**: Laser source

**2.**: Micrometer

**3.**: C-clamp

**4.**: Clamp stand

**5.**: White plane surface



As seen on *Figure-4*, the slit width *d* is controlled by a Vernier micrometer, with 1×10-5m being the smallest controllable unit. The micrometer is suspended at an appropriate height by a C-clamp attached to a clamp stand, and a spirit level is used in the process of positioning the micrometer to ensure that the slit is perpendicular to the ground. The laser source used is a Griffin class-II helium-neon laser box that has a constant output power of 1mW to control the variable *P*.

In addition to *Figure-4*, a straight pencil line is drawn right beneath and parallel to the slit of the micrometer that extends back to meet a protractor that is attached to the laser source to control the variable *θi* at 90°±0.5°. This measure is taken to ensure minimum distortion of the diffraction pattern produced. A meter ruler is also placed perpendicularly beneath the micrometer and extends to the white plane surface to control the variable *D* at 1 ± 0.005 m. To hold the apparatus more firmly in place, blu tack is applied between the C-clamp and the micrometer to prevent slipping; Pieces blu tack is also placed at the bottom of the clamp stand to secure the setup in place.

*3.2.2 Capacity of setup*

A test-run is conducted to determine the domain of slit widths that can be tested on this setup.

While the light source ceases to diffract at *d* = 3.70×10-4 ± 5×10-6 m, the projected diffraction pattern becomes unmeasurable at *d* = 5.0×10-5± 5×10-6 m. Hence the testable domain of slit width *d* on this setup is {3.70×10-4 m *≥ d ≥* 5.0×10-5m}.

*3.3 Measurement*

*3.3.1 Measuring w*

The width of the central maximum of each diffraction pattern is measured using a metal Vernier caliper that has a smallest unit of measurement of 0.0001m. To measure the width of the central maximum, the Vernier caliper is position flat against the plane surface above the diffraction pattern, and adjusted until the lower jaws aligns with the boundary between the central maximum and the first minima as seen on *Figure-5*.

*Figure-5*: Measuring the width of a diffraction pattern with a Vernier caliper

It must be noted that the transition from bright to dark fringes on a diffraction pattern is continuous, which makes the measurement of *w* subjective to some extent. To compensate for this effect, four trials of *w* measurements are taken for each distinct diffraction pattern, and the final results are compared with theoretical values to validate accuracy.

*3.3.2 Measuring I0*

The maximum intensity on each diffraction pattern is measured by using a digital Lux-meter that is composed of a light sensor connected to a LabQuest 2 computer interface. The smallest unit of measurement on this piece of apparatus is 1 Lux. To measure the maximum intensity, the sensor is slowly shifted across the diffraction pattern, while the intensity received by the sensor at each position is logged on a spreadsheet at the rate of 3 readings per second. The maximum value is then taken from the spreadsheet as the *I0* for the trial. Four trials of *I0* measurements are taken for each diffraction pattern to improve accuracy. To control the variable *La*, each trial of measurement for *I0* is always preceded by a measurement of the ambient luminosity by the same equipment to ensure *La* ≤2±1 Lux.

*3.3.3 Treatment of errors and uncertainties*

To simultaneously account for the range of uncertainty in repeated trials of raw measurements and the instrumental error of the measuring apparatus in individual measurements, an aggregate error is calculated for every measured trial average as follows:

In terms of propagating errors down into derived variables that are calculated from known values, we take the sum of all the relative errors associated with each value involved in the variable’s calculation and multiply it by the calculated value of the variable, which that:

For all tabulated results, uncertainty values are rounded to their first significant digit, while the values that they attach to are rounded to the same significant digit. Calculated theoretical values without uncertainties are to have the same number of significant digits as their corresponding raw measurements.

**4. Data Processing**

*4.1 Proximate Result*

*4.1.1* Relationship between *d* and *w*

*4.1.1.1* Theoretical value of *w*

The angle where the first minima occurs is calculated by the formula:

, where is the wavelength of the light source, and is the slit width. By applying [6] to *Figure-6*, it can be deduced that *w* is theoretically calculated by the equation:

*Figure-6*: Geometry of single-slit diffraction patterns

Given that the small angle approximation applies to the scenario on *Figure-6*, where , [7] can be simplified as:

Substituting in *D* = 1 m and into [8], we finally get:

All theoretical values of *w* tabulated on *Table-2* are calculated by [9].

*4.1.1.2* Experiment data

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| *Table-2*: Relationship between width of slit *d* and width of principle maximum *w* on the diffraction pattern | | | | | | | | |
| *d*: Width of slit (m)  *± 0.000005 m instrument error* | *w*: Measured width of diffraction pattern principle maximum (m) | | | | | | | |
| Trial 1 | Trial 2 | Trial 3 | Trial 4 | Trial average  *with aggregate error* | | | Calculated Theoretical Value |
| *± 0.00005 m instrument error* | | | |
| 0.000370 | 0.00340 | 0.00340 | 0.00330 | 0.00340 | 0.0034 | ± | 0.0001 | 0.00342 |
| 0.000360 | 0.00340 | 0.00350 | 0.00340 | 0.00340 | 0.0034 | ± | 0.0001 | 0.00352 |
| 0.000350 | 0.00340 | 0.00340 | 0.00350 | 0.00340 | 0.0034 | ± | 0.0001 | 0.00362 |
| 0.000340 | 0.00340 | 0.00340 | 0.00350 | 0.00350 | 0.0035 | ± | 0.0001 | 0.00372 |
| 0.000330 | 0.00340 | 0.00340 | 0.00350 | 0.00350 | 0.0035 | ± | 0.0001 | 0.00384 |
| 0.000320 | 0.00350 | 0.00350 | 0.00360 | 0.00350 | 0.0035 | ± | 0.0001 | 0.00396 |
| 0.000310 | 0.00350 | 0.00350 | 0.00360 | 0.00360 | 0.0036 | ± | 0.0001 | 0.00408 |
| 0.000300 | 0.00350 | 0.00340 | 0.00370 | 0.00380 | 0.0036 | ± | 0.0003 | 0.00422 |
| 0.000290 | 0.00400 | 0.00360 | 0.00380 | 0.00400 | 0.0039 | ± | 0.0003 | 0.00436 |
| 0.000280 | 0.00440 | 0.00420 | 0.00410 | 0.00430 | 0.0043 | ± | 0.0002 | 0.00452 |
| 0.000270 | 0.00440 | 0.00430 | 0.00420 | 0.00440 | 0.0043 | ± | 0.0002 | 0.00469 |
| 0.000260 | 0.00450 | 0.00420 | 0.00430 | 0.00440 | 0.0044 | ± | 0.0002 | 0.00487 |
| 0.000250 | 0.00460 | 0.00460 | 0.00450 | 0.00460 | 0.0046 | ± | 0.0001 | 0.00506 |
| 0.000240 | 0.00450 | 0.00490 | 0.00490 | 0.00460 | 0.0047 | ± | 0.0003 | 0.00527 |
| 0.000230 | 0.00510 | 0.00540 | 0.00510 | 0.00490 | 0.0051 | ± | 0.0003 | 0.00550 |
| 0.000220 | 0.00500 | 0.00570 | 0.00530 | 0.00520 | 0.0053 | ± | 0.0004 | 0.00575 |
| 0.000210 | 0.00520 | 0.00610 | 0.00530 | 0.00540 | 0.0055 | ± | 0.0005 | 0.00603 |
| 0.000200 | 0.00540 | 0.00620 | 0.00560 | 0.00570 | 0.0057 | ± | 0.0005 | 0.00633 |
| 0.000190 | 0.00580 | 0.00660 | 0.00580 | 0.00630 | 0.0061 | ± | 0.0005 | 0.00666 |
| 0.000180 | 0.00600 | 0.00720 | 0.00620 | 0.00680 | 0.0066 | ± | 0.0007 | 0.00703 |
| 0.000170 | 0.00640 | 0.00790 | 0.00650 | 0.00730 | 0.0070 | ± | 0.0008 | 0.00744 |
| 0.000160 | 0.00640 | 0.00810 | 0.00710 | 0.00770 | 0.0073 | ± | 0.0009 | 0.00791 |
| 0.000150 | 0.00710 | 0.00890 | 0.00740 | 0.00790 | 0.008 | ± | 0.001 | 0.00844 |
| 0.000140 | 0.00830 | 0.00920 | 0.00920 | 0.00820 | 0.0087 | ± | 0.0006 | 0.00904 |
| 0.000130 | 0.00800 | 0.00950 | 0.00960 | 0.00870 | 0.0090 | ± | 0.0009 | 0.00974 |
| 0.000120 | 0.00850 | 0.01010 | 0.01170 | 0.00920 | 0.010 | ± | 0.002 | 0.01055 |
| 0.000110 | 0.00920 | 0.01330 | 0.01290 | 0.01230 | 0.012 | ± | 0.002 | 0.01151 |
| 0.000100 | 0.01000 | 0.01120 | 0.01310 | 0.01290 | 0.012 | ± | 0.002 | 0.01266 |
| 0.000090 | 0.01170 | 0.01570 | 0.01560 | 0.01390 | 0.014 | ± | 0.002 | 0.01406 |
| 0.000080 | 0.01340 | 0.02370 | 0.01630 | 0.01430 | 0.017 | ± | 0.005 | 0.01582 |
| 0.000070 | 0.01590 | 0.01900 | 0.02110 | 0.01870 | 0.019 | ± | 0.003 | 0.01808 |
| 0.000060 | 0.02380 | 0.02290 | 0.02320 | 0.02140 | 0.023 | ± | 0.001 | 0.02109 |
| 0.000050 | 0.02580 | 0.03430 | 0.02730 | 0.02430 | 0.028 | ± | 0.005 | 0.02531 |

*Figure-7* shows that the measured values of *w* closely resemble the theoretical values calculated by [7], which largely lie within the acceptable range of errors and hence suggest that the measured values are valid and accurate. Since the first maxima will always pinpoint where the path difference of the diffracting light is , the negative relationship between *d* and *w* supports that the diffraction pattern becomes more dispersed in the *y* direction as the slit narrows.

*4.1.2* Relationship between *I0* and *w*

*4.1.2.1* Converting luminous intensity into light intensity

The lux-meter used in the experiment measures luminous intensity in Lux instead of light intensity in W m-2, hence luminous intensity must be converted into light intensity before proceeding with analysis. The conversion factor between Lux and W m-2, lm W-1, varies across the E-M spectrum, as light with different wavelength will have different brightness when emitted at the same frequency.

Although secondary research yielded no literature for the specific conversion factor at , a table of conversion factors at other wavelengths (*Georgia state university)* was found and attached to the appendix. The contents of the table is graphically plotted on *Figure-8*.

It can be deduced from *Figure-8* that the conversion factor between luminous intensity and light intensity is 165.4 lm W-1 at , hence we can say for the helium-neon laser used in the experiment that:

*4.1.2.2* Experiment data

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| *Table-3*: Relationship between width of slit *d* and maximum intensity *I0* on the diffraction pattern | | | | | | | | | | |
| *d*: Width of slit (m)  *± 0.000005 m*  *Instrument error* | *L0*: Measured maximum luminous intensity (Lux) | | | | | | | *I0*: Calculated maximum intensity  (W m-2)  *with propagated error* | | |
| Trial 1 | Trial 2 | Trial 3 | Trial 4 | Trial Average  *with aggregate error* | | |
| *± 1 Lux instrument error* | | | |
| 0.000370 | 8417 | 8460 | 8459 | 8432 | 8440 | ± | 20 | 51.0 | ± | 0.1 |
| 0.000360 | 8106 | 8222 | 8214 | 8245 | 8200 | ± | 70 | 49.6 | ± | 0.4 |
| 0.000350 | 7689 | 8119 | 7807 | 8007 | 8000 | ± | 200 | 48 | ± | 1 |
| 0.000340 | 7913 | 7934 | 7601 | 7932 | 7800 | ± | 200 | 47 | ± | 1 |
| 0.000330 | 7468 | 7567 | 7321 | 7568 | 7500 | ± | 100 | 45.2 | ± | 0.8 |
| 0.000320 | 7276 | 7432 | 7213 | 7211 | 7300 | ± | 100 | 44.0 | ± | 0.7 |
| 0.000310 | 6904 | 7024 | 6876 | 7094 | 7000 | ± | 100 | 42.2 | ± | 0.7 |
| 0.000300 | 6583 | 6678 | 6489 | 6632 | 6600 | ± | 100 | 39.9 | ± | 0.6 |
| 0.000290 | 6397 | 6432 | 6452 | 6546 | 6460 | ± | 80 | 39.0 | ± | 0.5 |
| 0.000280 | 6301 | 6331 | 6405 | 6387 | 6360 | ± | 50 | 38.4 | ± | 0.3 |
| 0.000270 | 5400 | 6027 | 6014 | 6034 | 5900 | ± | 300 | 35.5 | ± | 1.9 |
| 0.000260 | 5661 | 5788 | 5491 | 5512 | 5600 | ± | 200 | 33.9 | ± | 0.9 |
| 0.000250 | 5150 | 4952 | 4932 | 5209 | 5100 | ± | 100 | 30.6 | ± | 0.8 |
| 0.000240 | 4789 | 4885 | 4723 | 4976 | 4800 | ± | 100 | 29.3 | ± | 0.8 |
| 0.000230 | 4568 | 4671 | 4610 | 4764 | 4700 | ± | 100 | 28.1 | ± | 0.6 |
| 0.000220 | 4322 | 4321 | 4524 | 4298 | 4400 | ± | 100 | 26.4 | ± | 0.7 |
| 0.000210 | 3872 | 3726 | 3955 | 3867 | 3900 | ± | 100 | 23.3 | ± | 0.7 |
| 0.000200 | 3399 | 3628 | 3824 | 4043 | 3700 | ± | 300 | 23 | ± | 2 |
| 0.000190 | 3317 | 3440 | 3547 | 3671 | 3500 | ± | 200 | 21 | ± | 1 |
| 0.000180 | 3427 | 3013 | 3282 | 3222 | 3236 | ± | 208 | 20 | ± | 1 |
| 0.000170 | 3108 | 3000 | 3112 | 3007 | 3060 | ± | 60 | 18.5 | ± | 0.3 |
| 0.000160 | 2867 | 2816 | 2996 | 2706 | 2800 | ± | 100 | 17 | ± | 1 |
| 0.000150 | 2230 | 2533 | 2432 | 2671 | 2500 | ± | 200 | 15 | ± | 1 |
| 0.000140 | 2006 | 1991 | 1783 | 2028 | 2000 | ± | 100 | 12 | ± | 1 |
| 0.000130 | 1438 | 1756 | 1801 | 1875 | 1700 | ± | 200 | 10 | ± | 1 |
| 0.000120 | 1447 | 1658 | 1672 | 1704 | 1600 | ± | 100 | 9.8 | ± | 0.8 |
| 0.000110 | 1166 | 1232 | 1532 | 1324 | 1300 | ± | 200 | 8 | ± | 1 |
| 0.000100 | 807 | 1027 | 904 | 1098 | 1000 | ± | 100 | 5.8 | ± | 0.9 |
| 0.000090 | 567 | 623 | 589 | 576 | 590 | ± | 30 | 3.6 | ± | 0.2 |
| 0.000080 | 434 | 498 | 395 | 414 | 440 | ± | 50 | 2.6 | ± | 0.3 |
| 0.000070 | 288 | 256 | 291 | 264 | 280 | ± | 20 | 1.7 | ± | 0.1 |
| 0.000060 | 165 | 185 | 178 | 187 | 180 | ± | 10 | 1.1 | ± | 0.1 |
| 0.000050 | 50 | 52 | 54 | 53 | 52 | ± | 3 | 0.32 | ± | 0.02 |

Since the output power *P* of the helium-neon laser is rated at 1 mW constant, the output intensity of the light source is constant. In the context that the collimated beams produced by lasers have minimum dispersion, the positive relationship between *I0* and *d* seen from *Figure-9* also supports that the diffraction pattern becomes more dispersed as the slit narrows.

Fitting a linear trend line through the data set seems to show that there is a systematic error, as it does not pass through the origin where it theoretically should, as there will be no light passing through when the slit width is zero. However, *I0* at narrower slit widths deviates from the linear trend, where the rate of change gradually decreases and the data tends toward the origin. While there is no literature to refer to, it can’t be determined at this stage whether there is a systematic error or not, a possible reason for this observation is proposed under *Section-6.1* under evaluation.

*4.2 Deriving ultimate variables*

The uncertainty relation [1]**[[3]](#footnote-4)** between position *x* and momentum *p* is more formally known as:

*(University of New Mexico)*

, where the uncertainties and are defined by the standard deviations of position and momentum and . This shows that we can also derive and as the standard deviations of *y* and *py* if we can recreate the distribution of photons diffracting through the slit as functions of *y* and *py* from the proximate experiment results.

*4.2.1 Quantifying photons*

Quantifying photons is necessary for recreating the distribution of photons as a function of any variable. Since photons as quantum objects carry discrete quantities of energy, the most intuitive approach to quantify them is through energy.

The energy *E* of a single photon is calculated by the formula:

, where *c* is the speed of light, *h* is Planck’s constant and is the wavelength of the photon. Given the light source used in this experiment is monochromatic, which means that each photon emitted will always carry the same quantity of energy according to [12], it can be deduced that:

; Hence light intensity can be used as a unit to measure the quantity of photons when we model the distribution of the diffracting photons in the following sections.

*4.2.2 Deriving*

As the 1 mW helium-neon laser used in this experiment emits an uniform output, the distribution of the photons’ positions along the *y*-axis in the slit will follow a uniform distribution. Finding the standard deviation of the uniform distribution of the position of photons at each slit width will give the variable of .

For a uniform distribution spanning between the domain , the standard deviation of the distribution is calculated by the formula:

*(Weisstein)*

For as the standard deviation of the uniform distribution of *y*, the interval between *b* and *a* is the slit width *d*, hence:

All values tabulated on *Table-4* are calculated by using [15].

*4.2.3 Deriving*

To derive as the standard deviation of from the collected data, we must first establish a formula that can construct the distribution of diffracting photons as a function of based on the measured *w* and *I0* values. The distribution of diffracting photons as a function of can be modelled after Kirchhoff’s diffraction formula:

*(National University of Singapore)*

, which predicts the variation of light intensity *I* on any single-slit diffraction pattern as a function of angle given the slit width *d*, wavelength , and the maximum intensity *I0*.

Reviewing *Figure-3* in *Section-1.2* under introduction, it can be deduced that:

, where *p* is the magnitude of the photon’s resultant momentum. As the speed of light is constant in the same medium, the magnitude of a photon’s resultant momentum must be constant, and it is calculated by rearranging de Broglie’s hypothesized formula, [4]**[[4]](#footnote-5)**, as follows:

We know that for the helium-neon laser used in the experiment, hence substituting this value into [18] gives:

Substituting [19] into [17], we get:

Substituting [20] into [16], we get:

, which predicts the variation of light intensity on any single-slit diffraction pattern as a function of given the slit width *d* and the maximum intensity *I0*. It should be noted that from this point onward, intensity *I* represents the quantity photons travelling at a particular given [13]**[[5]](#footnote-6)**.

As the goal here is to construct the distribution of diffracting photons based on experiment data rather than predicting it, [21] must be further manipulated so that it takes the dependent variable *w* as input instead of the independent variable *d*.

Experiment confirms that the relationship between *d* and *w* is [8]**[[6]](#footnote-7)** at D=1, hence:

Substituting [22] into [21], we finally get:

, which can be used to construct the distribution of diffracting photons as a function of their momenta in the *y* direction given any pair of *w* and measurements made from the same diffraction pattern.

A more formal treatment of [23] resolves into the function:

, where denotes that intensity *I* is treated as a function rather than a variable. The domain of the function is restricted to the stated values because the magnitude of the component momentum cannot be greater than the magnitude of its resultant momentum *p*, which stays constant at .

Inputting each pair of measured average *w* and *I0* values from *Table-2* and *Table-3* into [24] produces the graph seen on *Figure-10*, which shows the variation of the distribution of diffracting photons as a function of their momentum in the *y* direction when the slit width changes. Alternatively, *Figure-10* also serves as a better visual representation of how the diffraction pattern becomes more dispersed when the slit narrows.

To calculate as the standard deviation of each of the distribution produced at different slit widths on *Figure-10*, we can consider the statistics formula for standard deviation:

, where *xi* stands for each of the value of the data, *fi* stands for the statistical frequency that *xi* appears, *n* is the total quantity of data, and *μ* is the mean of the data set.

If we substitute for *xi*, *fi* can be substituted by the function since light intensity can be regarded as a unit measuring the quantity of photons as previously discussed. Taking note that is a continuous function, the summation expression should be replaced by a definite integral with the domain of , where *n* as the total quantity of data becomes total area under the function . The following expression is obtained after the aforementioned changes are made to [25]:

as of [24] is an even function, hence ; Therefore the finalized equation for calculating is:

All values tabulated on *Table-4* are calculated by using [27].

*4.3 Relationship between and of diffracting photons*

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| *Table-4*: Relationship between uncertainty of position and uncertainty of momentum of diffracting photons | | | | | | | | | | |
| Independent Variables | | Dependent Variables | | | | | | | | |
| *d*: Width of slit (m)  *± 0.000005 m instrument error* | Δ*y*: Uncertainty of position of photons (m)  *± 1.44E-06 m* | *w*: Average width of principle maxima (m)  *with aggregate error* | | | *I0*: Average maximum intensity  (W m-2)  *with propagated error* | | | Δ*py*: Uncertainty of momentum in the *y* direction (kg m s-1)  *with propagated error* | | |
| 0.000370 | 0.000107 | 0.0034 | ± | 0.0001 | 51.0 | ± | 0.1 | 6.3E-30 | ± | 2E-31 |
| 0.000360 | 0.000104 | 0.0034 | ± | 0.0001 | 49.6 | ± | 0.4 | 6.1E-30 | ± | 2E-31 |
| 0.000350 | 0.000101 | 0.0034 | ± | 0.0001 | 48 | ± | 1 | 6.1E-30 | ± | 3E-31 |
| 0.000340 | 0.000098 | 0.0035 | ± | 0.0001 | 47 | ± | 1 | 6.5E-30 | ± | 3E-31 |
| 0.000330 | 0.000095 | 0.0035 | ± | 0.0001 | 45.2 | ± | 0.8 | 6.5E-30 | ± | 3E-31 |
| 0.000320 | 0.000092 | 0.0035 | ± | 0.0001 | 44.0 | ± | 0.7 | 6.5E-30 | ± | 3E-31 |
| 0.000310 | 0.000089 | 0.0036 | ± | 0.0001 | 42.2 | ± | 0.7 | 6.6E-30 | ± | 3E-31 |
| 0.000300 | 0.000087 | 0.0036 | ± | 0.0003 | 39.9 | ± | 0.6 | 7.0E-30 | ± | 6E-31 |
| 0.000290 | 0.000084 | 0.0039 | ± | 0.0003 | 39.0 | ± | 0.5 | 7.6E-30 | ± | 6E-31 |
| 0.000280 | 0.000081 | 0.0043 | ± | 0.0002 | 38.4 | ± | 0.3 | 8.8E-30 | ± | 5E-31 |
| 0.000270 | 0.000078 | 0.0043 | ± | 0.0002 | 35.5 | ± | 1.9 | 8.8E-30 | ± | 8E-31 |
| 0.000260 | 0.000075 | 0.0044 | ± | 0.0002 | 33.9 | ± | 0.9 | 9.0E-30 | ± | 7E-31 |
| 0.000250 | 0.000072 | 0.0046 | ± | 0.0001 | 30.6 | ± | 0.8 | 9.1E-30 | ± | 4E-31 |
| 0.000240 | 0.000069 | 0.0047 | ± | 0.0003 | 29.3 | ± | 0.8 | 9.6E-30 | ± | 8E-31 |
| 0.000230 | 0.000066 | 0.0051 | ± | 0.0003 | 28.1 | ± | 0.6 | 1.05E-29 | ± | 8E-31 |
| 0.000220 | 0.000064 | 0.0053 | ± | 0.0004 | 26.4 | ± | 0.7 | 1.1E-29 | ± | 1E-30 |
| 0.000210 | 0.000061 | 0.0055 | ± | 0.0005 | 23.3 | ± | 0.7 | 1.2E-29 | ± | 2E-30 |
| 0.000200 | 0.000058 | 0.0057 | ± | 0.0005 | 23 | ± | 2 | 1.2E-29 | ± | 2E-30 |
| 0.000190 | 0.000055 | 0.0061 | ± | 0.0005 | 21 | ± | 1 | 1.3E-29 | ± | 2E-30 |
| 0.000180 | 0.000052 | 0.0066 | ± | 0.0007 | 20 | ± | 1 | 1.4E-29 | ± | 2E-30 |
| 0.000170 | 0.000049 | 0.0070 | ± | 0.0008 | 18.5 | ± | 0.3 | 1.5E-29 | ± | 2E-30 |
| 0.000160 | 0.000046 | 0.0073 | ± | 0.0009 | 17 | ± | 1 | 1.5E-29 | ± | 3E-30 |
| 0.000150 | 0.000043 | 0.008 | ± | 0.001 | 15 | ± | 1 | 1.7E-29 | ± | 4E-30 |
| 0.000140 | 0.000040 | 0.0087 | ± | 0.0006 | 12 | ± | 1 | 1.8E-29 | ± | 2E-30 |
| 0.000130 | 0.000038 | 0.0090 | ± | 0.0009 | 10 | ± | 1 | 1.9E-29 | ± | 4E-30 |
| 0.000120 | 0.000035 | 0.010 | ± | 0.002 | 9.8 | ± | 0.8 | 2.1E-29 | ± | 5E-30 |
| 0.000110 | 0.000032 | 0.012 | ± | 0.002 | 8 | ± | 1 | 2.5E-29 | ± | 8E-30 |
| 0.000100 | 0.000029 | 0.012 | ± | 0.002 | 5.8 | ± | 0.9 | 2.4E-29 | ± | 7E-30 |
| 0.000090 | 0.000026 | 0.014 | ± | 0.002 | 3.6 | ± | 0.2 | 2.8E-29 | ± | 6E-30 |
| 0.000080 | 0.000023 | 0.017 | ± | 0.005 | 2.6 | ± | 0.3 | 3E-29 | ± | 1E-29 |
| 0.000070 | 0.000020 | 0.019 | ± | 0.003 | 1.7 | ± | 0.1 | 3.2E-29 | ± | 7E-30 |
| 0.000060 | 0.000017 | 0.023 | ± | 0.001 | 1.1 | ± | 0.1 | 4.0E-29 | ± | 5E-30 |
| 0.000050 | 0.000014 | 0.028 | ± | 0.005 | 0.32 | ± | 0.02 | 5E-29 | ± | 1E-29 |

While *Figure-11* shows the relationship between and , the gradient of the linear trend lines on *Figure-12* represents the value of the constant *k* in the anticipated relationship [2]**[[7]](#footnote-8)**, where *k* is the fundamental limit of the products of and of the diffracting photons.

The line of best-fit on *Figure-12* and *Figure-11* both have the equation:

, showing that the best-fit value of *k* for the dataset on *Table-4* is:

.

The margin of error for the possible range of *k* values that can be derived from *Figure-12* is determined by using the following formula:

, given:

Best-fit gradient *m =*

Maximum gradient *mmax =*

Minimum gradient *mmin =*

, which after rounding to one significant figure of error gives:

Hence, the experiment ultimately shows that

for photons diffracting through narrow slits of varying widths.

**5. Conclusion**

There is substantial evidence to support that the single slit diffraction of monochromatic coherent light does demonstrate Heisenberg’s uncertainty principle.

Firstly, [28] as the best-fit model on both *Figure-11* and *Figure-12* clearly shows an inversely proportional relationship between and , which strongly agrees with [2] as the hypothesized relationship. [28] deviates from each data point plotted on both *Figure-11* and *Figure-12* within the acceptable margin of error, while its coefficient of determination with the data points plotted on *Figure-11* is , which shows that it is both a valid and a relatively accurate model of the dataset from *Table-4*

Secondly, the experimental value of *k* derived from *Figure-12* as the fundamental limit of the products of and for diffracting photons strongly agrees with [5]**[[8]](#footnote-9)** as the literature relationship for diffracting electrons, which states that Planck’s constant *h* is the fundamental limit of the product between and . Noting that is equivalent to m2 kg s-1, the literature value of in [5] falls well within the margin of error given by [29], while the best-fit value of

derived from [28] only deviates from Planck’s constant by approximately 4.2%.

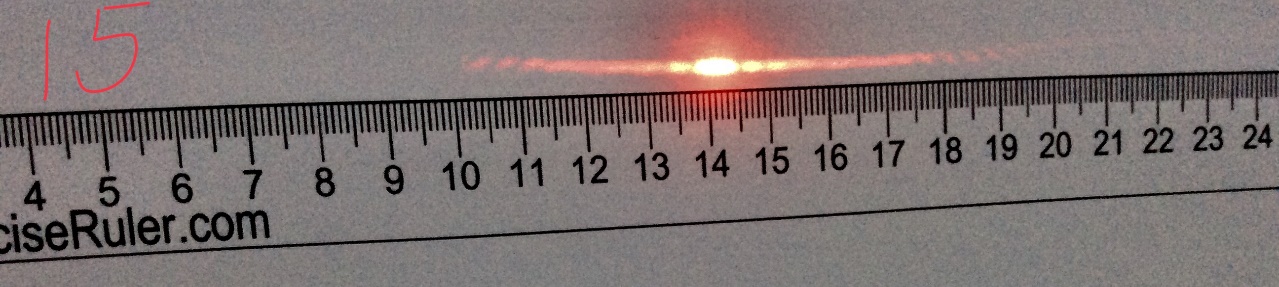
Therefore, it can be solidly concluded that the relationship between the width of a single slit and the dispersion of the diffraction pattern formed when monochromatic coherent light passes through it does demonstrate Heisenberg’s uncertainty principle.

**6. Evaluation**

*6.1 Limitations of experiment setup*

 A significant limitation of the experiment setup lies with using a micrometer to adjust the slit width. The micrometer used in the experiment changes the slit width through two metal rods, where one rod moves towards another when the slit width is restricted. As seen from *Figure-13*, this limitation makes it impossible for the slit to remain at the exact center of the laser dot, which causes the diffraction produced by the setup to be slightly asymmetrical in terms of light intensity as seen on *Figure-14.*

*Figure-13*: The mechanism of using a micrometer to control slit width



*Figure-14*: Photo of one of the diffraction pattern produced during the experiment, the right side is slightly brighter than the left side

With reference to *Figure-9*, this limitation is potentially what caused the rate of change of *I0* to decrease at narrower slit widths, where the slit becomes more and more off-center in terms of its position with the laser dot. If this were the case, there would be a systematic error for the measurement of *I0* in the experiment.

However, this limitation did not have a significant adverse effect on the reliability of the results of the experiment. Firstly, the affected variable *I0* can be cancelled out during the calculation for by isolating it as a constant from the integral expressions involving the function , which means that the proximate systematic error will not be carried forward to the ultimate dependent variable of . Secondly, *Figure-7* shows that the measured widths of the central maxima largely agrees with the literature values within the acceptable margin of error, which means that the overall geometry of the diffraction patterns remained relatively intact despite the flaw in the experiment setup.

This limitation can be readily resolved if more professional equipment, such as the Vernier diffraction apparatus (*Vernier software & technology)*, were available. The slit of the Vernier diffraction apparatus will always be positioned at the exact center of the light source, and hence will always produce a symmetrical diffraction pattern.

*6.3 Scope of further research*

One question remain unanswered by this investigation is whether photons exhibit the classical uncertainty inequality on a three-dimensional scale. Although it is not experimentally feasible to evaluate the three-dimensional uncertainty principle of diffracting photons, a meaningful extension to this investigation would be to experimentally derive the two-dimensional uncertainty principle of diffracting photons by passing laser through a circular aperture instead of a slit. This way we can further confirm that the diffraction of light is caused by photons obeying Heisenberg’s uncertainty principle in two dimensions, and potentially extrapolate on the three-dimensional uncertainty relation based on the results obtained for the uncertainty relations in one and two dimensions.

3989 words

excluding figures, tables, equations and annotations.

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**Appendix**

Table including different conversion factors between luminous intensity and light intensity at varying wavelengths *(Georgia State University, Luminous Efficacy Tables)*

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Wavelength λ (nm) | Photopic Luminous Efficacy Vλ | Photopic Conversion lm/W | Scotopic Luminous Efficacy V'λ | Scotopic Conversion lm/W |
| 380 | 0.000039 | 0.027 | 0.000589 | 1.001 |
| 390 | 0.000120 | 0.082 | 0.002209 | 3.755 |
| 390 | 0.000120 | 0.082 | 0.002209 | 3.755 |
| 400 | 0.000396 | 0.270 | 0.009290 | 15.793 |
| 410 | 0.001210 | 0.826 | 0.034840 | 59.228 |
| 420 | 0.004000 | 2.732 | 0.096600 | 164.220 |
| 430 | 0.011600 | 7.923 | 0.199800 | 339.660 |
| 440 | 0.023000 | 15.709 | 0.328100 | 557.770 |
| 450 | 0.038000 | 25.954 | 0.455000 | 773.500 |
| 460 | 0.060000 | 40.980 | 0.567000 | 963.900 |
| 470 | 0.090980 | 62.139 | 0.676000 | 1149.200 |
| 480 | 0.139020 | 94.951 | 0.793000 | 1348.100 |
| 490 | 0.208020 | 142.078 | 0.904000 | 1536.800 |
| 500 | 0.323000 | 220.609 | 0.982000 | 1669.400 |
| 507 | 0.444310 | 303.464 | 1.000000 | 1700.000 |
| 510 | 0.503000 | 343.549 | 0.997000 | 1694.900 |
| 520 | 0.710000 | 484.930 | 0.935000 | 1589.500 |
| 530 | 0.862000 | 588.746 | 0.811000 | 1378.700 |
| 540 | 0.954000 | 651.582 | 0.655000 | 1105.000 |
| 550 | 0.994950 | 679.551 | 0.481000 | 817.700 |
| 555 | 1.000000 | 683.000 | 0.402000 | 683.000 |
| 560 | 0.995000 | 679.585 | 0.328800 | 558.960 |
| 570 | 0.952000 | 650.216 | 0.207600 | 352.920 |
| 580 | 0.870000 | 594.210 | 0.121200 | 206.040 |
| 590 | 0.757000 | 517.031 | 0.065500 | 111.350 |
| 600 | 0.631000 | 430.973 | 0.033150 | 56.355 |
| 610 | 0.503000 | 343.549 | 0.015930 | 27.081 |
| 620 | 0.381000 | 260.223 | 0.007370 | 12.529 |
| 630 | 0.265000 | 180.995 | 0.003335 | 5.670 |
| 640 | 0.175000 | 119.525 | 0.001497 | 2.545 |
| 650 | 0.107000 | 73.081 | 0.000677 | 1.151 |
| 660 | 0.061000 | 41.663 | 0.000313 | 0.532 |
| 670 | 0.032000 | 21.856 | 0.000148 | 0.252 |
| 680 | 0.017000 | 11.611 | 0.000072 | 0.122 |
| 690 | 0.008210 | 5.607 | 0.000035 | 0.060 |
| 700 | 0.004102 | 2.802 | 0.000018 | 0.030 |
| 710 | 0.002091 | 1.428 | 0.000009 | 0.016 |
| 720 | 0.001047 | 0.715 | 0.000005 | 0.008 |
| 730 | 0.000520 | 0.355 | 0.000003 | 0.004 |
| 740 | 0.000249 | 0.170 | 0.000001 | 0.002 |
| 750 | 0.000120 | 0.082 | 0.000001 | 0.001 |
| 760 | 0.000060 | 0.041 | 0.000000 | 0.000 |
| 770 | 0.000030 | 0.020 | 0.000000 | 0.000 |

1. [1]: page 5 [↑](#footnote-ref-2)
2. [8]: page 15 [↑](#footnote-ref-3)
3. [1]: page 5 [↑](#footnote-ref-4)
4. [4]: page 9 [↑](#footnote-ref-5)
5. [13]: page 21 [↑](#footnote-ref-6)
6. [8]: page 15 [↑](#footnote-ref-7)
7. [2]: page 8 [↑](#footnote-ref-8)
8. [5]: page 9 [↑](#footnote-ref-9)